Aircraft Design for Mission Performance Using Nonlinear Multiobjective Optimization Methods

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A new technique that converts a constrained optimization problem to an unconstrained one where conflicting figures of merit may be simultaneously considered has been combined with a complex mission analysis system. The method is compared with existing single and multiobjective optimization methods. A primary benefit from this new method for multiobjective optimization is the elimination of separate optimizations for each objective, which is required by some optimization methods. A typical wide-body transport aircraft is used for the comparative studies.

Introduction

AIRCRAFT conceptual design is the process of determining an aircraft configuration that satisfies a set of mission requirements. Engineers within several diverse disciplines including but not limited to mass properties, aerodynamics, propulsion, structures, and economics perform iterative parametric evaluations until a design is developed. Convention limits each discipline to a subset of configuration parameters, subject to a subset of design constraints, and typically, each discipline has a different figure of merit.

Advanced design methods have been built into synthesis systems such that communication between disciplines is automated to decrease design time.[1,2] Each discipline may select its own set of design goals and constraints resulting in a set of thumbprint and/or carpet plots from which a best design may be selected. In addition, the conceptual design problem has been demonstrated to be very amenable to the use of formal mathematical programming methods, and these algorithms have been implemented to identify feasible designs quickly.[3-5]

The purpose of this paper is to investigate the use of multiobjective optimization methods for conceptual aircraft design where conflicting figures of merit are considered simultaneously. Three multiobjective methods[6-8] have been combined with a complex mission analysis system.[5] Tradeoffs of the methods are compared with single objective results. In addition, parametric results of the design space are presented. The aircraft chosen for this investigation is a typical wide-body transport.

General Multiobjective Optimization

The constrained multiobjective optimization problem stated in conventional formulation is to minimize

$$F_k(X), \qquad k = 1 \text{ to number of objectives} \tag{1}$$

such that

$$g_j(X) \le 0, \qquad j = 1 \text{ to number of constraints}$$

and

$$x_i^l \le x_i \le x_i^u, \qquad i = 1 \text{ to number of design variables}$$

where

$$X = (x_1, x_2, x_3, \ldots, x_n)^T, \qquad n = \text{number of design variables}$$

The fundamental problem is to formulate a definition of $F_k(X)$, the objective vector, when its components have different units of measure thereby reducing the problem to a single objective. Several techniques have been devised to approach this problem.[7] The methods selected for study in this paper transform the vector of objectives into a scalar function of the design variables. The constrained minimum for this function has the property that one or more constraints will be active and that any deviation from it will cause at least one of the components of the objective function vector to depart from its minimum, the classic Pareto-minimal solution.[9,10] One should add that multiobjective optimization results are expected to vary depending on the method of choice since the conversion method to a single scalar objective is not unique.

Formulation of the Mission/Performance Optimization Problem

The purpose of the optimization is to rapidly identify a feasible design to perform specific mission requirements, where several conflicting objectives and constraints are considered. The aircraft type selected for this study is a typical wide-body transport (Fig. 1) in the 22,680 kg weight class.[11] The aircraft has three high-bypass ratio turbofan engines, with

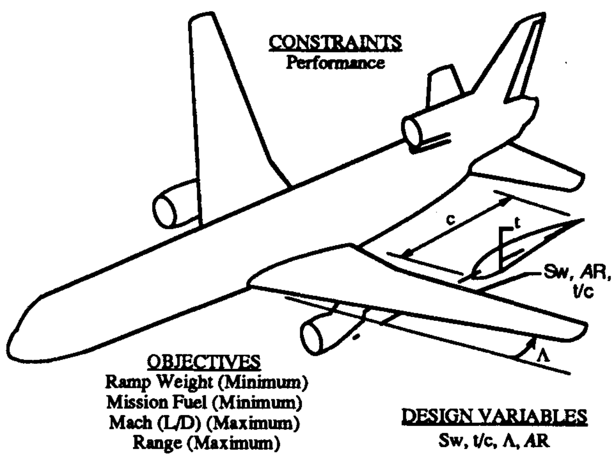

Fig. 1 Wide-body transport.

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6915 N thrust each. The mission requirements are design range 7413.0 km; cruise Mach number 0.83; cruise altitude 11.9 km; payload 42185.0 kg; and number of passengers and crew 256.0. The primary and reserve mission profiles are shown in Fig. 2.

The design variables considered (Fig. 1) are aspect ratio AR, area S_w, quarter chord sweep Λ, and thickness-to-chord ratio t/c of the wing, where the initial values chosen for all cases are

$$X_0 = \begin{Bmatrix} AR \\ S_w \\ \Lambda \\ t/c \end{Bmatrix} = \begin{Bmatrix} 11.0 \\ 361.0 \text{ m}^2 \\ 35.0 \text{ deg} \\ 0.11 \end{Bmatrix}$$

The objectives to be minimized or maximized for this investigation include

$F_1(X)$ = ramp weight (minimize)

$F_2(X)$ = mission fuel (minimize)

$F_3(X)$ = lift-to-drag ratio at constant cruise Mach number (maximize)

$F_4(X)$ = range with fixed ramp weight (maximize)

The functions to be maximized were formulated as negative values so that they could be used with a minimization algorithm. These objectives are first optimized for feasible single objective designs. The objectives are then considered simultaneously for multiobjective designs. Tables 1 and 2 list 14 cases, six multiobjective and eight single objective, along with the unconstrained objective function formulation used for each.

Each of the three formulations uses the Davidon-Fletcher-Powell variable metric optimization method to compute the search direction for finding a local unconstrained minimum of a function of many variables.[12]

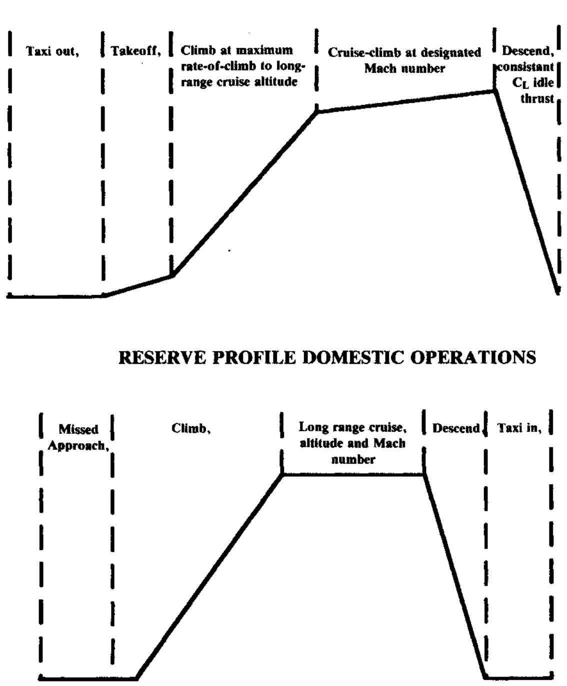

Fig. 2 Mission profile.

The inequality behavioral constraints used in each case are

$g_1(X)$ = lower limit on range, (1853.2 km)

$g_2(X)$ = upper limit on approach speed, (280.0 km/h)

$g_3(X)$ = upper limit on takeoff field length, (2700.0 m)

$g_4(X)$ = upper limit on landing field length, (2700.0 m)

$g_5(X)$ = lower limit on missed approach climb gradient thrust, (3458.0 N)

$g_6(X)$ = lower limit on second segment climb gradient thrust, (3458.0 N)

$g_7(X)$ = upper limit on mission fuel capacity (fuel capacity of wing plus fuselage)

Table 1 Multiobjective cases

Case number	KSOPT multiobjectives
1	$F_1(X)$ and $F_2(X)$
2	$F_1(X)$ and $F_2(X)$ and $F_3(X)$
	Penalty method weighted composite multiobjectives
3	$F_1(X)+F_2(X)$
4	$F_1(X)+F_2(X)+10{,}000\,F_3(X)$
	Global criterion method target objectives
5	$F_1^T(X)=201{,}629$ kg
	$F_2^T(X)=60{,}954$ kg
6	$F_1^T(X)=201{,}629$ kg
	$F_2^T(X)=60{,}954$ kg
	$F_3^T(X)=M\,(28.1)$

Table 2 Single objective cases

Case number	KSOPT single objectives
7	$F_1(X)$
8	$F_2(X)$
9	$F_3(X)$
10	$F_4(X)$
	Penalty method single objectives
11	$F_1(X)$
12	$F_2(X)$
13	$F_3(X)$
14	$F_4(X)$

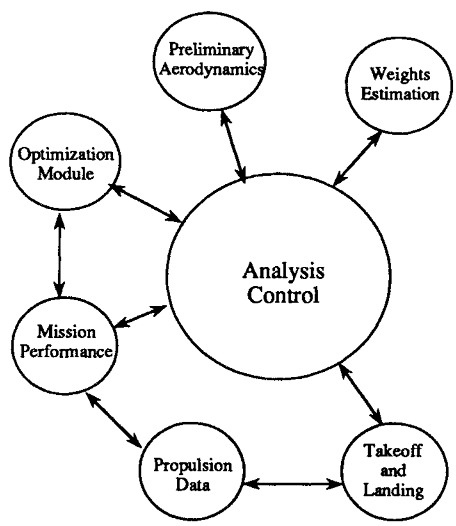

Fig. 3 FLOPS primary modules.

Table 3 Single objective design results

	X_0 initial conditions	Final values mission fuel, minimize: Case 8 KSOPT	Final values mission fuel, minimize: Case 12 PF	Final values ramp weight, minimize: Case 7 KSOPT	Final values ramp weight, minimize: Case 11 PF	Final values Mach L/D, maximize: Case 9 KSOPT	Final values Mach L/D, maximize: Case 13 PF	Final values range, maximize: Case 10 KSOPT	Final values range, maximize: Case 14 PF
Design variables									
AR, x_1	11.00	18.20	18.94	11.35	11.10	22.13	22.14	10.68	10.39
Sw, x_2, m^2	361.0	304.0	295.0	281.1	281.4	381.0	361.0	331.0	361.0
Sweep, x_3, deg	35.00	26.16	27.62	22.00	22.22	30.21	36.39	22.00	22.22
t/c, x_4	0.11	0.091	0.0913	0.0996	0.0989	0.087	0.107	0.099	0.098
Objective functions									
Ramp weight, $F_1(X)$, kg	207729.0	219248.0	220155.0	201629.0	201763.0	256156.0	239332.0	219248.0	219248.0
Mission fuel, $F_2(X)$, kg	67136.0	60954.0	60728.0	66891.0	66981.0	62791.0	62882.0	79492.0	79040.0
$M(L/D)$, $F_3(X)$	0.83 (19.34)	0.83 (24.50)	0.83 (24.76)	0.83 (18.92)	0.83 (18.78)	0.83 (28.09)	0.83 (22.86)	0.83 (19.25)	0.83 (19.31)
Range, $F_4(X)$, km	7413.0	7413.0	7413.0	7413.0	7413.0	7413.0	7413.0	8974.0	8922.0
Constraints									
g_1	—	−1.0	−1.0	−1.0	−1.0	−1.0	−1.0	−0.210	−0.203
g_2	—	−0.0307	−0.0140	−0.0327	−0.0332	−0.0636	−0.0699	−0.0701	−0.0910
g_3	—	−0.0601	−0.0181	−0.114	−0.129	−0.410	−0.0781	−0.112	−0.165
g_4	—	−0.0227	−0.00166	−0.0326	−0.444	−0.0479	−0.0628	−0.0626	−0.0870
g_5	—	−0.568	−0.554	−0.00227	−0.532	−0.565	−0.601	−0.391	−0.403
g_6	—	−0.459	−0.451	−0.217	−0.211	−0.475	−0.509	−0.110	−0.112
g_7	—	−0.0249	−0.000754	−0.0293	−0.0266	−0.102	−0.114	−0.126	−0.0635
Other quantities									
Span, (b), m	63.0	74.3	70.6	56.5	56.5	91.8	89.36	59.40	61.2
L/D	19.34	24.50	24.76	18.92	18.78	28.09	22.86	19.25	19.31
W/S	117.80	147.60	152.70	147.00	146.80	137.70	135.9	136.00	129.80
T/W	0.327	0.318	0.309	0.337	0.337	0.280	0.284	0.310	0.310
Function evaluations	—	483	255	158	146	213	242	190	180

where the constraint functions g_j are written in terms of computable functions stated as demand (X) and capacity. These functions provide the measure of what a design can sustain vs what it is asked to carry:

$$g_j(X) = \text{demand}(X)/\text{capacity} - 1 \tag{2}$$

In addition, side constraints were imposed on wing sweep and wing area in the form of upper and lower bounds.

Description of the Analysis System for Mission Performance

The Flight Optimization System (FLOPS) is an aircraft configuration optimization system developed for use in conceptual design of a new transport and fighter aircraft and the assessment of advanced technology.[5] The system is a computer program consisting of four primary modules shown in Fig. 3: weights, aerodynamics, mission performance, and takeoff and landing. The weights module uses statistical data from existing aircraft which were curve fit to form empirical wing weight equations using an optimization program. The transport data base includes aircraft from the small business jet to the jumbo jet class. Aerodynamic drag polars are generated using the empirical drag estimation technique[13] in the aerodynamics module. The mission analysis module uses weight, aerodynamic data, and an engine deck to calculate performance. Based on energy considerations, an optimum climb profile is flown to the start of the cruise condition. The cruise segment may be flown for maximum range with ramp weight requirements specified; optimum Mach number for maximum endurance; minimum mission fuel requirements; and minimum ramp weight requirements. Takeoff and landing analyses include ground effects, while computing takeoff and landing field lengths to meet Federal Air Regulation (FAR) obstacle clearance requirements.

Description of Objective Function Formulation Methods

Envelope Function Formulation, KSOPT

This algorithm is a new technique for converting a constrained optimization problem to an unconstrained one[6] and is easily adaptable for multiobjective optimization.[14] The conversion technique replaces the constraint and objective function boundaries in n-dimensional space with a single surface. The method is based on a continually differentiable function[15]

$$\text{KS}(X) = \frac{1}{\rho} \log_e \sum_{k=1}^{K} e^{f_k(X)} \tag{3}$$

where $f_k(X)$ is a set of K objective and constraint functions, and ρ controls the distance of the KS function surface from the maximum value of this set of functions evaluated at X. Typical values of ρ range from 5 to 200. The KS function defines an envelope surface in n-dimensional space representing the influence of all constraints and objectives of the mission analysis problem. The initial design may begin from a feasible or infeasible region.

Global Criterion Formulation

The optimum design is found by minimizing the normalized sum of the squares of the relative difference of the objective

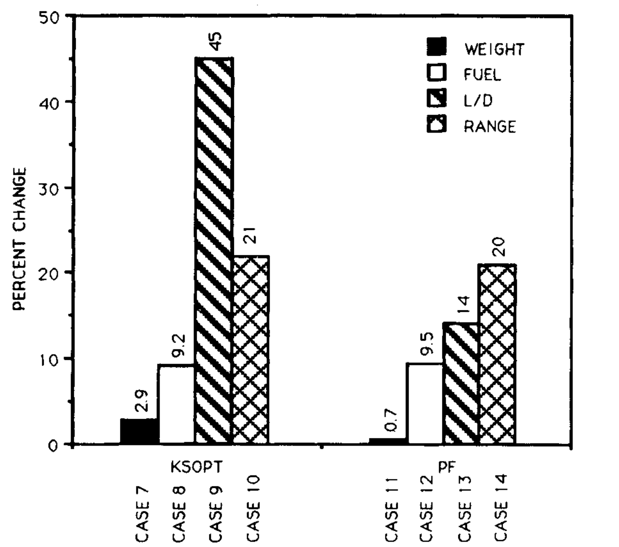

Fig. 4 Single objective optimization change from initial conditions.

functions. Single objective solutions are first obtained and are referred to as fixed-target objectives. Computed values then attempt to match the fixed-target objectives. Written in the generalized form

$$F^*(X) = \sum_{k=1}^{K} \left[\frac{F_k^T(X) - F_k(X)}{F_k^T(X)} \right]^2 \tag{4}$$

where F_k^T is the target value of the kth objective and F_k is the computed value. F^* is the Global Criterion (GC) performance function.[7] The performance function F^* was then minimized using the KSOPT formulation described previously.

Utility Function Formulation Using a Penalty Function Method

The optimum design is found by minimizing a utility function stated as

$$F^*(X) = \sum_{k=1}^{K} w_k F_k(X) \tag{5}$$

where w_k is a designer's choice weighting factor for the kth objective function F_k to be minimized. This composite objective function is included in a quadratic extended interior penalty function (PF).[16] This function is stated in generalized form as

$$\tilde{F}(x,r_p) = F^*(X) - r_p \sum_{j=1}^{m} G_j(X) \tag{6}$$

and

$$G_j(X) = \begin{cases} \dfrac{1}{g_j(X)} & \text{for } g_j(X) \geq \epsilon \\[2ex] \dfrac{2\epsilon - g_j(X)}{\epsilon^2} & \text{for } g_j(X) < \epsilon \end{cases}$$

where the $r_p \sum_{j=1}^{m} G_j(X)$ term penalizes $\tilde{F}(X,r_p)$, the performance function in proportion to the amount by which the constraints are violated, and ϵ is a designer's choice transition parameter. The value of the penalty multiplier r_p is initially estimated based on the type of problem to be solved and is varied during the optimization process. The penalty multiplier r_p is made successively smaller to arrive at a constrained minimum.

Results and Discussion

Single Objective Function Optimization

Single objective results for two of the methods are presented, the envelope function KSOPT and the classic PF methods. Single objective cases were run to establish a baseline for comparison of multiobjective performance. In addition, target objectives are obtained for the GC method. Final optimization values are presented in Table 3 for both methods. Both techniques converged to very similar designs for all cases listed in Table 2. Greatest modifications from the initial design are seen in lift-to-drag ratio L/D, cases 9 and 13, and range, cases 10 and 14.

Lift-to-drag was modified by increasing the aspect ratio and wing area, thus minimizing the wing loading W/S. Thrust requirements T/W increased due to the larger ramp weight. In addition, the wing was made thinner and unswept. The KSOPT method converged to a 23% higher L/D vs the PF method. This is typically due to the way constraint boundaries are followed.

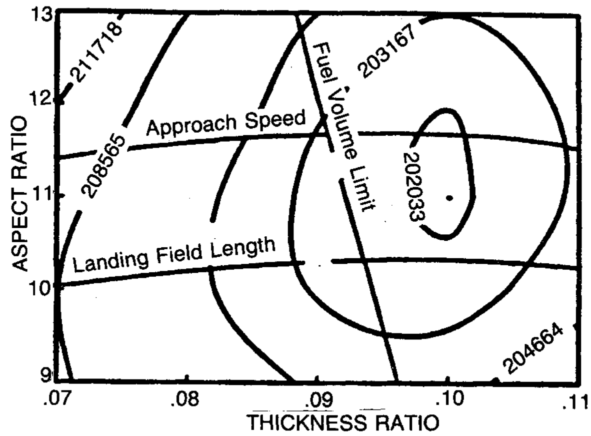

Fig. 5 Ramp weight as a function of aspect ratio and thickness ratio.

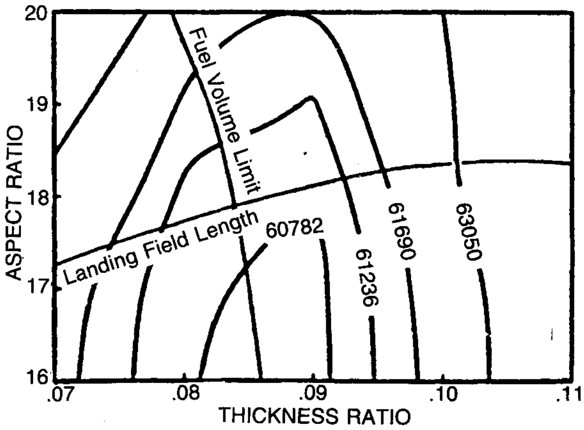

Fig. 6 Mission fuel as a function of aspect ratio and thickness ratio.

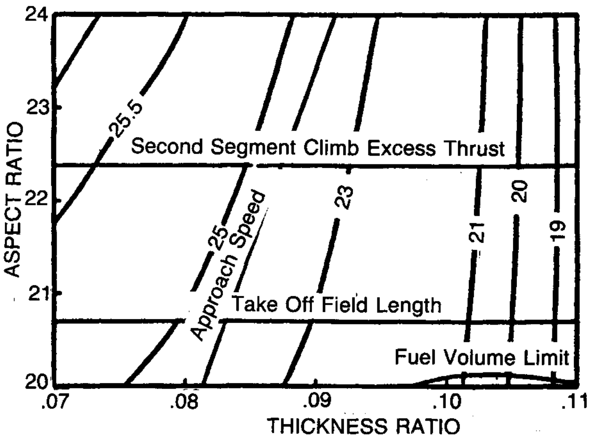

Fig. 7 Mach(L/D)as a function of aspect ratio and thickness ratio.

Range improvements, cases 10 and 14, were accomplished by unsweeping the wing to the lower limit allowed, and wing volume was adjusted to carry the maximum fuel load with reserves at the penalty of increased ramp weight. In addition, the optimizers reduced wing thickness, area, and aspect ratio from initial values. Wing loading was kept at a minimum. KSOPT again produced a slightly better design compared with the PF method.

To minimize mission fuel requirements, cases 8 and 12, the aspect ratio was increased and the wing area was decreased. In addition, the wing was unswept and made thinner. This design improved aerodynamic performance by over 20% from the initial value while ramp weight increased slightly. The PF method converged to a slightly better design for this case.

Ramp weight, cases 7 and 11, was decreased by unsweeping the wing to the lower limit of 22 deg. The aspect ratio is essentially unchanged from the initial condition design point. The wing thickness was decreased, along with a decrease in area. Aerodynamic performance was not penalized significantly from the initial design value; KSOPT produced a slightly lower ramp weight.

The chart in Fig. 4 compares the final design objective's percent change from the initial design point.

Parametric Results of the Design Space

Point designs, obtained parametrically, for minimum ramp weight, minimum mission fuel, and maximum Mach L/D are shown in Figs. 5–7. Wing aspect ratio and thickness-to-chord

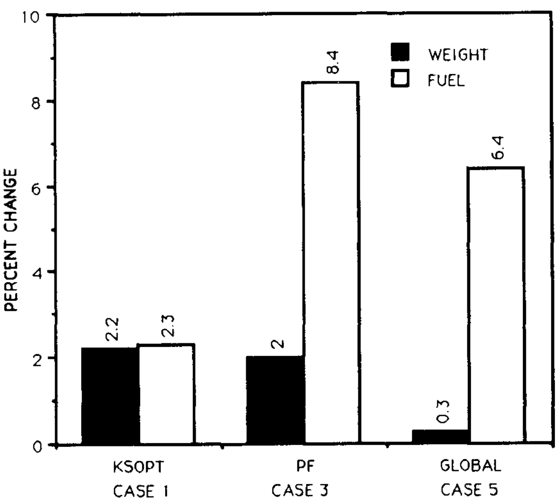

Fig. 8 Two objective optimization compromise from single objective cases.

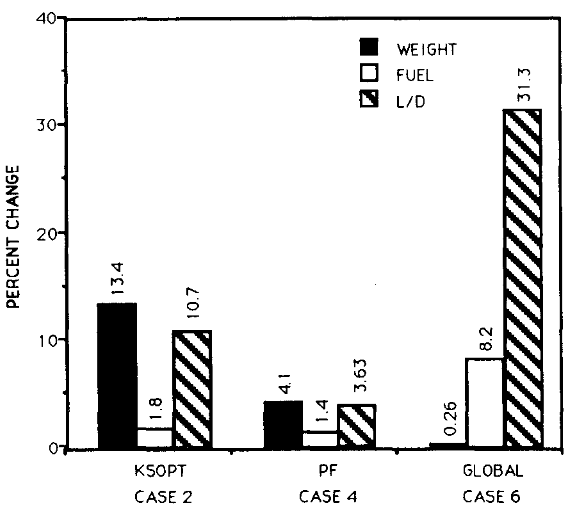

Fig. 9 Three objective optimization compromise from single objective cases.

Table 4 Two objective design results

	Ramp weight and mission fuel, minimize		
	Case 1 KSOPT	Case 3 PF	Case 5 GC
	Design variables		
AR, x_1	14.51	10.28	12.31
Sw, x_2, m^2	289.0	369.0	282.0
Sweep, x_3, deg	24.50	22.17	22.00
t/c, x_4	0.0948	0.0946	0.0958
	Objective functions		
Ramp weight, $F_1(X)$, kg	206268.0	205499.0	202360.0
Mission fuel, $F_2(X)$, kg	62353.0	65803.0	64647.0
$M(L/D)$, $F_3(X)$	0.83(21.77)	0.83(19.84)	0.83(19.97)
Range, $F_4(X)$, km	7413.0	7413.0	7413.0
	Constraints		
g_1	−1.0	−1.0	−1.0
g_2	−0.0352	−0.148	−0.0334
g_3	−0.116	−0.349	−0.120
g_4	−0.0334	−0.161	−0.0334
g_5	−0.537	−0.529	−0.481
g_6	−0.385	−0.263	−0.281
g_7	−0.0341	−0.254	−0.0243
	Other quantities		
Span, (b), m	64.7	58.9	55.5
L/D	21.77	19.84	19.97
W/S	146.20	114.0	146.80
T/W	0.329	0.331	0.336
Function evaluations	325	121	98

Table 5 Three objective design results

	Ramp weight and mission fuel, minimize and M (L/D), maximize		
	Case 2 KSOPT	Case 4 PF	Case 6 GC
	Design variables		
AR, x_1	16.87	15.49	11.643
Sw, x_2, m^2	365.0	291.0	286.0
Sweep, x_3, deg	26.39	22.12	24.20
t/c, x_4	0.083	0.089	0.099
	Objective functions		
Ramp weight, $F_1(X)$, kg	228716.0	210065.0	202162.0
Mission fuel, $F_2(X)$, kg	62041.0	61564.0	65980.0
$M(L/D)$, $F_3(X)$	0.83(25.09)	0.83(22.81)	0.83(19.29)
Range,F_4 (X), km	7413.0	7413.0	7413.0
	Constraints		
g_1	−1.0	−1.0	−1.0
g_2	−0.0956	−0.0305	−0.0405
g_3	−0.171	−0.0921	−0.134
g_4	−0.0961	−0.0267	−0.0416
g_5	−0.599	−0.543	−0.466
g_6	−0.465	−0.402	−0.249
g_7	−0.118	−0.0839	−0.0485
	Other quantities		
Span, (b), m	78.4	63.4	54.4
L/D	25.09	22.08	19.29
W/S	128.50	147.60	144.60
T/W	0.297	0.323	0.336
Function evaluations	62	174	73

Table 6 Best single objective results

Case	Objective	Method	Final value
12	Fuel	PF	60728.0 kg
7	Weight	KSOPT	201629.0 kg
9	L/D	KSOPT	28.09

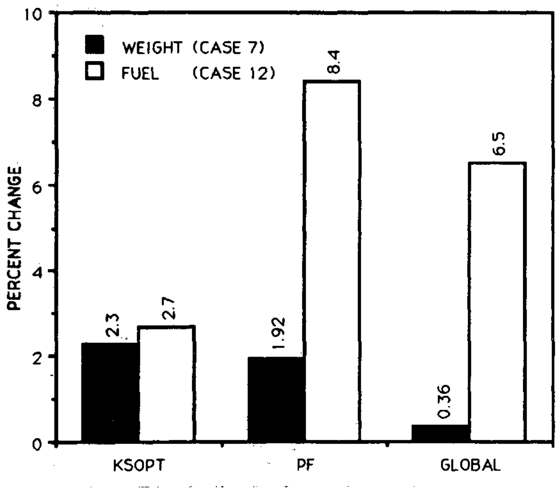

Fig. 10 Two objective optimization compromise from best single objective cases.

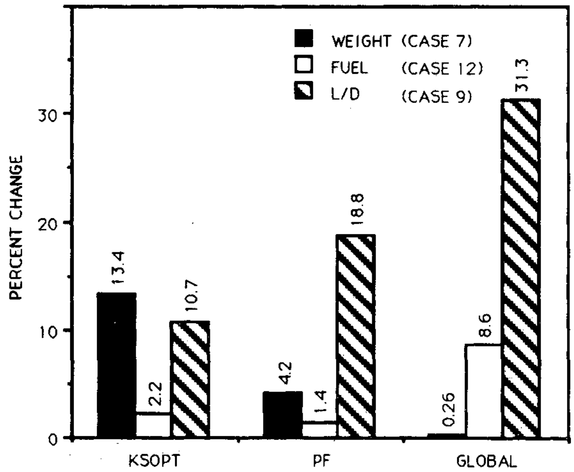

Fig. 11 Three objective optimization compromise from best single objective cases.

ratio were varied, whereas other design variables were set to optimum values given in Table 3, cases 8, 7, and 9, respectively. The design space is shown with the most critical constraints or criteria governing the design. To arrive at the optimum point designs shown by traditional parametric trade studies, over 256 evaluations would have been required.

Multiobjective Optimization

Multiobjective optimization considers all conflicting design objectives and constraints simultaneously to meet mission specifications. Three methods are compared: the envelope function KSOPT, the PF method, and the GC method. Feasible designs were obtained for two objectives (Table 4), and three objectives (Table 5) satisfying all constraints.

Comparison of Two Objective with Single Objective Design

Figure 8 shows the percent deviation or compromise from each method's single objective design. KSOPT treated ramp weight and mission fuel equally whereas the PF and GC methods favored ramp weight, preferring to pay a larger penalty for mission fuel. This behavior is expected with the PF and GC methods since the ramp weight is larger in magnitude giving this objective greater influence. This effect could have been eliminated by judicious normalization or weighting.

Comparison of Three Objective with Single Objective Design

Figure 9 shows the percent deviation or compromise from each method's single objective design. KSOPT traded aerodynamic efficiency L/D and ramp weight to keep fuel requirements down. The PF method weighted L/D to a greater extent since the weighting coefficient w_k was 10,000, with small penalties in ramp weight and mission fuel. The GC penalty behavior is similar to the two objective results in that the ramp weight was weighted more over mission fuel and aerodynamic efficiency. The overall compromise is lowest for the PF method.

Comparison with Overall Best Single Objective Designs

The best single objective design results are listed in Table 6 along with objectives and methods. Since L/D was not part of the objective function set (Fig. 10), two objective compromised results were very similar to Fig. 8. Three objectives (Fig. 11) caused the design space to be more constrained. KSOPT again traded ramp weight and L/D to keep mission fuel requirements down. The PF method traded in a similar way but compromised L/D to a greater extent. The GC method gave more priority to ramp weight because of its magnitude. The overall compromise of KSOPT and PF was about the same at 26.3 and 24.4%, respectively, and the GC method 40.4%.

Conclusions

A typical wide-body subsonic transport aircraft configuration was used to investigate the use of three multiobjective optimization methods: 1) an envelope of constraints and objectives, KSOPT, 2) a penalty function, and 3) the global criterion. The methods were coupled with a complex mission performance analysis system. The optimizer used with all three methods is the Davidon-Fletcher-Powell variable metric method for unconstrained optimization. Multiobjective compromised solutions were obtained for two and three objective functions. Feasible designs for each objective were also obtained using single objective optimization as well. The initial value design variable vector X_0 and the constraints g_1–g_7 were the same for all cases in this comparative study.

The KSOPT method was able to follow constraint boundaries closely and considered the influence of all constraints and objectives in a single continuously differentiable envelope function. KSOPT defines the optimum such that the function component with the greatest relative slope dominates the solution. The PF method also produced feasible designs similar to the KSOPT final designs for single objective optimization. This method, however, weights the individual objective functions in the multiobjective cases.

The GC method is usually applied to multiobjective problems but may be used in the single objective problem if a target objective is supplied. This would be equivalent to imposing an upper or lower bound on the performance function. The GC method has a disadvantage in resource requirements, requiring separate single objective optimizations to provide target objectives.

Computational effort has been measured in functional evaluations, shown in the tables of results. They are defined as the number of calls to the analysis procedures from the optimization procedures. Function evaluations are very similar for single objective cases except for mission fuel using KSOPT. This deviation is due to the methods implementation, convergence criteria, and the way constraint boundaries are followed. The multiobjective table shows the GC method with the least functional evaluations; however, single objective function evaluations must be included with these values, thereby making it the most costly in terms of number of analyses.

All of the methods produced feasible solutions within the design space. Attributes of the methods, such as ease of use, data requirements, and programming should also be considered when evaluating their performance along with computational efficiency. Many cases have been compared, too numerous to report herein, where initial design variables were changed up to 40% above and below the initial values given in this report. KSOPT continued to perform in a robust manner compared to the PF method, producing similar final designs within 1% of the mean. Based on the results of this study and the preceding considerations, KSOPT is thus concluded to be a viable general method for multiobjective optimization. Finally, one should add that multiobjective optimization results are expected to vary depending on the method of choice.

Acknowledgment

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